

Quantum-gravity phenomenology with gamma rays and UHE cosmic rays¹

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ABSTRACT

In recent years several ideas for experimental searches of effects induced by quantum properties of space-time have been discussed. Some of these ideas concern the role in quantum spacetime of the ordinary Lorentz symmetry of classical flat spacetime. Deviations from ordinary (classical) Lorentz symmetry are now believed to be rather natural in non-commutative space-times, models based on String Theory and models based on Loop Quantum Gravity. Observations of gamma rays and ultra-high-energy cosmic rays could play a key role in the development of this research programme.

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1 Introduction

Quantum-Gravity Phenomenology[1] is an intentionally vague name for a new approach to research on the possible non-classical (quantum) properties of spacetime.

This approach does not adopt a specific formalism for the description of the short-distance structure of spacetime (*e.g.*, “string theory”, “loop quantum gravity” and “noncommutative geometry” are seen as equally deserving mathematical-physics programmes); it is rather the proposal that quantum-gravity research should proceed just in the familiar old-fashioned way: through small incremental steps starting from what we know, combining mathematical-physics studies with experimental studies to reach deeper and deeper layers of understanding of the short-distance structure of spacetime. For various “historical” reasons (mostly connected with the lack of guidance from experiments) research on quantum gravity has wondered off this traditional strategy: the most popular quantum-gravity approaches, such as string theory and loop quantum gravity, could be described as “top-to-bottom approaches”, since they start off with some key assumption about the structure of spacetime at the Planck scale and then they try (with limited, vanishingly small, success) to work their way back to “reality”, the realm of doable experiments. With “quantum-gravity phenomenology” I would like to refer to all studies that are somehow related with the “bottom-to-top approach”, consistently with traditional strategy of physics research.

Since the problem at hand is extremely difficult (arguably the most challenging problem ever faced by the physics community) it appears likely that the two complementary approaches might combine in a useful way: for the “bottom-to-top approach” it is important to get some guidance from the (however tentative) indications emerging from the “top-to-bottom approaches”, while for “top-to-bottom approaches” it might be very useful to be alerted by quantum-gravity phenomenologists with respect to the type of new effects that could be most effectively tested experimentally².

Until very recently the idea of a quantum-gravity phenomenology, and in particular of attempts of identification of experiments with promising sensitivity, was very far from the main interests of quantum-gravity research. One isolated idea had been circulating from the mid 1980s: it had been realized[2, 3, 4] that the sensitivity of CPT tests using the neutral-kaon system is such that even small effects of CPT violation originating at the Planck scale³ might in principle be revealed. These pioneering works on CPT tests were for more than a decade the only narrow context in which the implications of quantum gravity were being discussed in relation with experiments, but over the last 4 years several new ideas for tests of Planck-scale physics have appeared at increasingly fast pace, leading me to argue[1, 5] that the times might be right for a larger overall effort in this direction, which indeed could be called “quantum-gravity phenomenology”. At the present time (in addition to the already mentioned CPT tests) there are several examples of experimentally accessible contexts in which conjectured quantum-gravity effects are being considered, including studies of in-vacuo dispersion using gamma-ray astrophysics[6, 7], studies of laser-interferometric limits on quantum-gravity induced distance fluctuations[8, 9], studies of the role of the Planck length in the determination of the energy-momentum-conservation threshold conditions for certain particle-physics processes[10, 11, 12, 13], and studies of the role of the Planck length in the determination of particle-decay amplitudes[14]. These experimental/phenomenological studies might represent the cornerstones of quantum-gravity phenomenology since they are as close as one can

²It is hard for “top-to-bottom approaches” to obtain a complete description of low-energy physics, but perhaps it would be possible to dig out predictions on some specific spacetime features that appear to deserve special attention in light of the corresponding experimental sensitivities.

³The possibility of Planck-scale-induced violations of the CPT symmetry has been extensively considered in the literature. One simple point in support of this possibility comes from the fact that the CPT theorem, which holds in our present conventional theories, relies on exact locality, whereas in quantum gravity it appears plausible to assume lack of locality at Planckian scales.

get to direct tests of space-time properties, such as space-time symmetries. Other experimental proposals that should be seen as part of the quantum-gravity-phenomenology programme rely on the mediation of some dynamical theory in quantum space-time; comments on these other proposals can be found in Refs.[1, 15, 16, 17, 18, 19].

In these lecture notes I intend to emphasize those aspects of quantum-gravity-phenomenology that are relevant for the astrophysics community. The relevant topic is the one that concerns the faith of the Lorentz symmetry of classical spacetime when the spacetime is quantized. Since the Lorentz symmetry of classical flat (Minkowski) spacetime is verified experimentally to very high accuracy, it appears that any deviation from classical Lorentz symmetry, which might emerge from quantum-gravity theories, would be subject to severe experimental constraints. As a result Lorentz-symmetry tests are a key component of the programme of “quantum-gravity phenomenology”[1, 20, 21].

My main focus here will be on the faith of Lorentz invariance at the quantum-spacetime level. A large research effort has been devoted to this subject. Most of these studies focus on the possibility that Lorentz symmetry might be “broken” (in a sense clarified later in these notes) at the quantum level; however, I have recently shown that Lorentz invariance might be affected by spacetime quantization in a softer manner: there might be no net loss of symmetries but the structure of the Lorentz transformations might be affected by the quantization procedure[22, 23]. In the following I shall describe rather pedagogically the main differences between the broken-symmetry and my new deformed-symmetry scenario. In addition I will comment on the type of astrophysical observations, involving gamma rays and ultra-high-energy cosmic rays, which could provide evidence of such symmetry-related quantum properties of space-time. An exciting recent development in this area is that certain puzzling gamma-ray and UHE cosmic-ray observations are being actively discussed as possible first manifestations of a quantum property of space-time.

Before going forward with these main points on my agenda for these lecture notes, let me make a parenthetical remark, further claryfing the objectives of quantum-gravity phenomenology: The primary challenge of quantum-gravity phenomenology is the one of establishing the properties of space-time at Planckian distance scales, since most theoretical arguments suggest that this is the characteristic scale of quantum space-time effects. However, there is also recent discussion of the possibility that quantum-spacetime effects might be stronger than usually expected, *i.e.* with a characteristic energy scale that is much smaller (perhaps in the TeV range!) than the Planck energy. Examples of mechanisms leading to this possibility are found in string-theory models with large extra dimensions[24] and in certain noncommutative-geometry models[25]. Of course, the study of the phenomenology of these models is in the spirit of quantum-gravity phenomenology, but it is, in a sense, to be considered as a sideline development (and it is less challenging than the quantum-gravity-phenomenology efforts that pertain effects originating genuinely at the Planck scale).

2 The faith of Lorentz symmetry in quantum space-time

If Nature hosts some form of “quantization” (even just in the general weak sense of “non-classical” properties) of space-time, this of course would also apply to flat spacetimes (*e.g.* if spacetime is in general discrete or noncommutative then of course the particular case of flat spacetime will also be described in the same way). One might argue (more or less convincingly) that quantum effects should be stronger in strong-curvature contexts, such as the ones involving black holes, but our capability of detailed experimental studies of such contexts is vanishingly small. Instead, in certain flat-spacetime contexts our experiments reach extremely high precision and therefore even relatively small effects induced by quantum properties of spacetime might be detectable. This is one of the key strategic points of my view on the development of quantum-gravity phenomenology[1, 5].

In flat quantum spacetimes a key characteristic is the role of the Planck length, L_p . If the Planck length only has the role we presently attribute to it, which is basically the role of a coupling constant (an appropriately rescaled version of the gravitational coupling G), no problem arises for FitzGerald-Lorentz contraction, but if we try to promote L_p to the status of an intrinsic characteristic of space-time structure (or a characteristic of the kinematic rules that govern particle propagation in space-time) it is nearly automatic to find conflicts with FitzGerald-Lorentz contraction[22, 23].

For example, it is very hard (perhaps even impossible) to construct discretized versions or non-commutative versions of Minkowski space-time which enjoy ordinary Lorentz symmetry. Pedagogical illustrative examples of this observation have been discussed, *e.g.*, in Ref.[26] for the case of discretization and in Refs.[27, 28] for the case of non-commutativity. The action of ordinary (classical) boosts on discretization length scales (or non-commutativity length scales) will naturally be such that different inertial observers would attribute different values to these lengths scales, just as one would expect from the mechanism of FitzGerald-Lorentz contraction.

There are also dynamical mechanisms (of the spontaneous symmetry-breaking type) that can lead to deviations from ordinary Lorentz invariance; it appears for example that this might be possible in String Field Theory[29].

Both in String Theory and in Loop Quantum Gravity⁴ it is also natural to consider certain external-field backgrounds, which, in the appropriate sense[22, 23] (they provide a way to identify a preferred class of inertial observers), break Lorentz invariance.

Departures from ordinary Lorentz invariance are therefore rather plausible at the quantum-gravity level. Here I want to emphasize that there are at least two possibilities: (i) Lorentz invariance is broken and (ii) Lorentz invariance is deformed.

2.1 Deformed Lorentz invariance

In order to be specific about the differences between deformed and broken Lorentz invariance let me focus on the dispersion relation $E(p)$ which will naturally be modified in either case. Let me also assume, for the moment, that the deformation be Planck-length induced: $E^2 = m^2 + p^2 + f(p, m; L_p)$. If the function f is nonvanishing and nontrivial and the energy-momentum transformation rules are ordinary (the ordinary Lorentz transformations) then clearly f cannot have the exact same structure for all inertial observers. In this case one would speak of an instance in which Lorentz invariance is broken. If instead f does have the exact same structure for all inertial observers, then necessarily the transformations between

⁴As I shall argue more carefully elsewhere[30], in Loop Quantum Gravity there might even be a fundamental departure from classical Lorentz invariance. This can be deduced from studies arguing that Loop Quantum Gravity predicts a fixed discrete spectrum of area eigenvalues, independently of the characteristic scale of curvature of the surface whose area is being measured (and therefore also for flat surfaces in flat spacetimes). One of the primary implications of Lorentz invariance is that the same experiment is seen by different observers in different ways which are however predictably (classically) connected by Lorentz transformations. If, for example, a series of measurements by one observer all give the same result of an area measurement, say the result A_0 , then according to classical Lorentz invariance those same measurements should be seen by another observer as measurements all giving the same but different, say A_1 , result (with A_1 related to A_0 by the appropriate boost). When the spectrum of the area of a flat surface in a flat spacetime is discrete this property of classical Lorentz invariance is at risk: the results A_0 being all the same would reflect the fact that one is dealing with what is an area eigenstate for observer O_0 , and A_0 should be an eigenvalue of the area operator, but, if the second observer O_1 is only minutely boosted with respect to O_0 , one should find that A_1 , the boosted value of A_0 , could not possibly be another eigenvalue (if the boost is small enough it will not be sufficient for reaching another eigenvalue in the discrete list of eigenvalues that composes the spectrum of the area operator) and it would be paradoxical for observer O_1 to find systematically repeated measurement results A_1 .

these observers must be deformed. In this case one would speak of an instance in which the Lorentz transformations are deformed, but Lorentz invariance is preserved (in the deformed sense).

While much work has been devoted to the case in which Lorentz invariance is actually broken, the possibility that Lorentz invariance might be deformed was introduced only very recently by this author[22, 31, 32, 23, 33]. An example in which all details of the deformed Lorentz symmetry have been worked out is the one in which one enforces as an observer-independent statement the dispersion relation

$$L_p^{-2} \left(e^{L_p E} + e^{-L_p E} - 2 \right) - \vec{p}^2 e^{-L_p E} = m^2 \quad (1)$$

In leading (low-energy) order this takes the form

$$E^2 - \vec{p}^2 + L_p E \vec{p}^2 = m^2 . \quad (2)$$

The Lorentz transformations and the energy-momentum conservation rules are accordingly modified[22, 23, 33].

2.2 Broken Lorentz invariance

The case of broken Lorentz invariance requires fewer comments since it is more familiar to the community. In preparation for the following sections it is useful to emphasize that the same dispersion relation (2), which was shown in Refs.[22, 23] to be implementable as an observer-independent dispersion relation in a deformed-symmetry scenario, can also be considered[6] as a characteristic dispersion relation of a broken-symmetry scenario. In this broken symmetry scenario the dispersion relation (2) would still be valid but only for one “preferred” class of inertial observers (*e.g.* the natural CMBR frame) and it would be valid approximately in all frames not highly boosted with respect to the preferred frame. In highly-boosted frames one might find the same form of the dispersion relation but with different value of the deformation scale (different from L_p). All this follows from the fact that in the broken-symmetry scenario the laws of transformation between inertial observers are unmodified. Accordingly also energy-momentum conservation rules are unmodified.

Another scenario in which one finds broken Lorentz invariance is the one of canonical noncommutative spacetime, in which the dispersion relation is modified (with different deformation term[34, 35]), but, again, the energy-momentum Lorentz transformation rules are not modified. This example of noncommutative spacetime has been recently shown to be relevant for the description of string theory in certain external-field backgrounds (see, *e.g.*, Ref.[25, 34]).

3 Illustrative example: photon-pair pion decay

Before discussing the role that observations of gamma rays and UHE cosmic rays could play in the development of this research area, let me clarify, in this Section, that the differences between the broken-symmetry and the deformed-symmetry case can be very significant for what concerns experimental signatures. This is also important since it proves that the relevant astrophysics observations might not only provide us the first manifestation of a quantum space-time property: they might even distinguish between different quantum pictures of spacetime.

In order to render very explicit the differences between the broken-symmetry and the deformed-symmetry case I consider here the simplest example in which these differences are rather dramatic: photon-pair pion decay. I adopt in one case deformed energy-momentum

conservation[23], as required by the deformed Lorentz transformations of the deformed-symmetry case, while in the other case I adopt ordinary energy-momentum conservation, as required by the fact that the Lorentz transformation rules are unmodified in the broken-symmetry case, but for both cases I impose the same dispersion relation (2).

In the broken-symmetry case, combining (2) with ordinary energy-momentum conservation rules, one can establish a relation between the energy E_π of the incoming pion, the opening angle θ between the outgoing photons and the energy E_γ of one of the photons (the energy of the second photon is of course not independent; it is given by the difference between the energy of the pion and the energy of the first photon):

$$\cos(\theta) = \frac{2E_\gamma E'_\gamma - m_\pi^2 + 3L_p E_\pi E_\gamma E'_\gamma}{2E_\gamma E'_\gamma + L_p E_\pi E_\gamma E'_\gamma}, \quad (3)$$

where indeed $E'_\gamma \equiv E_\pi - E_\gamma$. This relation shows that at high energies (starting at values of E_π of order $(m_\pi^2/L_p)^{1/3}$) the phase space available to the decay is anomalously reduced: for given value of E_π certain values of E_γ that would normally be accessible to the decay are no longer accessible (they would require $\cos\theta > 1$).

In the deformed-symmetry case one enforces the deformed conservation rules[23]

$$E_\pi = E_\gamma + E'_\gamma, \quad \vec{p}_\pi = \vec{p}_\gamma + \vec{p}_{\gamma'} + L_p E_\gamma \vec{p}_{\gamma'}, \quad (4)$$

which, when combined again with (2), give rise to the different relation

$$\cos(\theta) = \frac{2E_\gamma E'_\gamma - m_\pi^2 + 3L_p E_\gamma^2 E'_\gamma + L_p E_\gamma E_\gamma'^2}{2E_\gamma E'_\gamma + 3L_p E_\gamma^2 E'_\gamma + L_p E_\gamma E_\gamma'^2}. \quad (5)$$

Here it is easy to check that one is never led to consider the paradoxical condition $\cos\theta > 1$. There is therefore no severe implication of the deformed-symmetry case for the amount of phase space available for the decays (certainly not at energies around $(m_\pi^2/L_p)^{1/3}$, possibly at Planckian energies).

4 An agenda for gamma-ray and UHECR studies

The key points for the phenomenology of quantum-gravity-induced deviations from classical Lorentz invariance are possible deformations of the dispersion relation and possible deformations of the energy-momentum conservation conditions.

Whether or not there is an accompanying deformation of energy-momentum conservation⁵ a deformation of the dispersion relation is expected to give rise to in vacuo dispersion[6, 1, 7] and, possibly (if the space-time has corresponding structure[35]), to birefringence. In vacuo dispersion would provide a striking signature: the speed of massless particles would depend on wavelength⁶ and therefore photons that we somehow know to have been emitted simultaneously up to $\Delta_0 T$ precision would reach us with relative time delays $\Delta_1 T$, where $\Delta_1 T > \Delta_0 T$, and one should also find some dependence of $\Delta_1 T$ on the amount of time

⁵In the case of deformation of Lorentz symmetry both the dispersion relation and the energy-momentum conservation conditions are modified simultaneously, since they both must reflect[22, 23] the structure of the deformed transformation rules between inertial observers.

⁶The ordinary dispersion relation is linear for massless particles, and therefore dE/dp is wavelength (energy) independent. A nonlinear Planck-length-deformed dispersion relation will instead inevitably lead to wavelength-dependent dE/dp .

the photon spent travelling in space-time (*i.e.* time spent under the influence of quantum properties of space-time). As discussed in Refs.[1, 6, 7] this type of effect can be naturally studied in the context of observations of gamma-ray bursts and observations of the high-energy photons emitted by certain blazars, such as Mk421. Certain gamma-ray observatories soon to be operational will have excellent sensitivity toward this type of effect, and in particular GLAST[36] is planning dedicated studies. Interest in such studies is also growing in AMS[37].

As discussed in the previous Section, also certain aspects of particle-decay physics, at high energies, may carry an important trace of quantum-space-time effects. In that context however the implications of a dispersion-relation deformation do depend strongly on whether there is an associated deformation of energy-momentum conservation (*i.e.* depend on whether one is dealing with a scenario with deformed symmetries or instead one is dealing with a scenario with broken symmetries). The outlook of these studies based on particle-decay anomalies is described in Ref.[14], also using a related data analysis reported in Ref.[38].

But perhaps the most powerful tool for the experimental investigation of quantum-gravity-induced deviations from ordinary Lorentz invariance is provided by “threshold anomalies”[13]. It is to this topic, which deserves being discussed in detail, that I devote the remainder of the Section. It is intriguing to notice that the prediction of these threshold anomalies appears to be consistent with some puzzling results of astrophysics observations. In two different regimes, UHECRs and multi-TeV photons, the universe appears to be more transparent than expected. UHECRs should interact with the Cosmic Microwave Background Radiation (CMBR) and produce pions. TeV photons should interact with the Infra Red (IR) photons and produce electron-positron pairs. These interactions should make observations of UHECRs with $E > 5 \cdot 10^{19} \text{eV}$ (the GZK limit)[39] or of gamma-rays with $E > 10 \text{TeV}$ [40] from distant sources unlikely. Still UHECRs above the GZK limit and Mk501 photons with energies up to 24 TeV are observed.

A CMBR photon and a UHE proton with $E > 5 \cdot 10^{19} \text{eV}$ should satisfy the kinematic requirements (threshold) for pion production. UHE protons should therefore lose energy, due to photopion production, and should slow down until their energy is below the GZK energy. At higher energies the proton’s mean free path decreases rapidly and it is down to a few Mpc at $3 \cdot 10^{20} \text{eV}$. Yet more than 15 CRs have been observed with nominal energies at or above $10^{20} \pm 30\% \text{eV}$ [41]. There are no astrophysical sources capable of accelerating particles to such energies within a few tens of Mpc from us. Furthermore, if the CRs are produced homogeneously in space and time, we would expect a break in the CR spectrum around the GZK threshold, which is not seen.

HEGRA has detected high-energy photons with a spectrum ranging up to 24 TeV[42] from Markarian 501 (Mk501), a BL Lac object at a redshift of 0.034 ($\sim 157 \text{Mpc}$). This observation indicates a second paradox of a similar nature. A high energy photon with energy E can interact with an IR background photon with wavelength $\lambda \sim 30 \mu\text{m}(E/10 \text{TeV})$ and produce an electron-positron pair. The mean free path of TeV photons depends on the spectrum of the corresponding IR background. Recent data from DIRBE[43, 44, 45] and from ISOCOM [46] suggest that the mean free path for 20TeV photons should be much shorter than the one of 10TeV photons. However, no apparent break is seen in the spectrum of Mk501 in the region 10-20TeV.

The UHECR paradox is well established. Numerous theoretical models, mostly requiring new physics, have been proposed for its resolution (see Ref.[47] for a recent review). With much less data, and with some uncertainty on the IR background, the Mk501 TeV-photon paradox is less established. However, if indeed this must be considered as a paradox, there are no models for its resolution, apart from the possibility that the IR background estimates are too large. Planned experiments will soon provide us better data on both issues. At present it appears reasonable to assume, just as a working hypothesis, that both paradoxes are real.

The interpretation of these paradoxes as threshold anomalies is appealing for several reasons. In both paradoxes low-energy photons interact with high energy particles. The relevant reactions should take place at a kinematic threshold. In both cases the center-of-mass threshold energies are rather modest and the physical processes involved are well tested and understood. In spite of these similarities, so far, there is no model that explains both paradoxes within a single theoretical scheme (unless the model accommodates an irritatingly large number of parameters). This appears to provide encouragement for the idea that quantum-gravity-induced deviations from ordinary Lorentz invariance might be responsible for both paradoxes⁷.

In order to illustrate the mechanism of threshold anomalies, let us consider, for example, the broken-symmetry case already considered in the preceding section. I will now apply it to the kinematics of the process of electron-positron pair production, which is relevant for the Mk501 paradox. Combining (2) with ordinary energy-momentum conservation rules, one can establish that at threshold the energy E of the hard Mk501 photon and the energy ϵ of the soft background photon must satisfy the relation $E \simeq m_e^2/\epsilon + L_p m_e^6/(8\epsilon^4)$. The correction $L_p m_e^6/(8\epsilon^4)$ is indeed sufficient to push the threshold energy upwards by a few TeV, consistently with the observations. As shown in Refs.[13] (and references therein), an analogous result holds for the photopion threshold, which is relevant for the cosmic-ray paradox.

This type of analysis provides encouragement (of course, very preliminary) for the hypothesis that the two paradoxes might be the first ever manifestation of a quantum (Planck-length related) property of spacetime.

Just like in the case of pion decay, considered in the preceding Section, also for the evaluation of threshold anomalies there are large quantitative differences (which will be discussed in detail in a paper now in preparation[50]) between the case in which Lorentz symmetry is broken and the case in which Lorentz symmetry is deformed. More accurate information on the paradoxes, such as the one that will be provided by Auger[51], can therefore even start pointing us toward the proper language for the description of the short-distance (quantum) structure of spacetime.

Experimental studies such as the ones planned by Auger will also in general clarify whether the origin of the paradoxes is indeed kinematical. I want to stress that, in this respect, it is important to get high-quality data in the neighbourhood of the expected GZK cutoff, perhaps even more than establishing how far (how high in energy) the cosmic-ray flux extends. In fact, the kinematical mechanism of threshold anomalies leads to the definite general prediction that nothing at all particular should happen at the GZK scale, since the GZK threshold is simply moved forward (or eliminated all together[13, 52]) by the deviations from classical Lorentz invariance. Other attempts of explaining the cosmic-ray paradox instead must coexist with the GZK threshold and therefore (unless huge parametric fine-tuning is allowed) will inevitably predict at least some peculiarity to occur at the GZK scale.

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⁷It is of course also possible to consider deviations from Lorentz invariance that do not have quantum-gravity origin[48, 49], but, as discussed in Refs.[13], the idea of a quantum-gravity origin, besides being conceptually appealing, leads to a natural estimate for the magnitude of the effects, and this estimate appears to fit well the observations (while non-quantum-gravity approaches host a large number of parameters to be freely adjusted to obtain the needed magnitude of the departure from Lorentz-invariance).

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